Effective Permittivity in Tunable Microstrip and Coplanar Lines

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Abstract – The paper presents analysis of effective parameters of micro-mechanically tunable microstrip and coplanar lines. Calculations with proposed quasi-static method are compared against commercial electromagnetic problems simulation tools. Effect of loss decrease is demonstrated in transmission lines with micro-mechanical tuning.

Keywords – microstrip transmission line; coplanar transmission line; micromechanical tuning; effective permittivity

I. INTRODUCTION

Microstrip and coplanar transmission lines belong to most widely used elements of microwave circuits. They are used as connection waveguides between oscillators, amplifiers, antennas and so on. Some passive components, such as resonator based filters use sections of transmission lines as coupling elements.

Traditionally, transmission lines remained fixed elements, whose properties were assigned at design time. However, transmission lines with variable characteristics may serve as a basis for wide class of tunable devices. Mechanical reconfiguration is one of the ways to control propagation characteristics in operation time. For instance, displacement of transmission line parts may have significant influence on its properties [1]. Considerable changes may occur with displacements of few tens micrometers, which can be achieved with modern fast actuators, such as piezoelectric, electrostrictive and microelectromechanical systems. To achieve such high sensitivity variable discontinuity should be perpendicular to the field lines. Microstrip and coplanar lines suit well this purpose [2–4]. Examples of possible implementation are shown in Fig. 1.

Simulation and design of tunable lines is simplified with complex effective permittivity concept. This paper presents study of effective permittivity, and especially effective loss in microstrip and coplanar lines with movable conductors.

II. EFFECTIVE PERMITTIVITY CONCEPT

When conductor is lifted above substrate surface, the line becomes irregular along wave propagation direction. Accurate calculation of electromagnetic field distribution problem might be difficult task, so numerical techniques should be applied.

Electromagnetic problem can be solved using electric and magnetic scalar \( \psi^e, \psi^m \) and vector \( \mathbf{A}^e, \mathbf{A}^m \) potentials:

\[
\mathbf{E} = -i\omega \mathbf{A}^e - \nabla \psi^e; \quad \mathbf{H} = \frac{\nabla \times \mathbf{A}^e}{\mu_0};
\]

\[
\mathbf{E} = \frac{\nabla \times \mathbf{A}^m}{\varepsilon \varepsilon_0}; \quad \mathbf{H} = -i\omega \mathbf{A}^m - \nabla \psi^m.
\]

Using these potentials one can introduce electromagnetic filed distribution types with one of components being zero. Applying Lorentz’s calibration in the absence of external currents and free charges we derive:

\[
\nabla \left( \varepsilon \nabla \psi^e \right) + \varepsilon^2 \mu_0 \frac{\omega^2}{c^2} \psi^e = 0. \tag{1}
\]

In case of axial symmetry solution of (1) may be presented in the form:

\[
\psi^e(x,y,z) = \psi(x,y)Z(z),
\]

where \( \psi(x,y) \) is distribution of scalar potential in Oxy plane, \( Z(z) \) is distribution along propagation direction Oz. Then (1) splits in two equations with two mentioned distribution functions. In most practical cases electric field component along direction of propagation is much smaller and could be neglected. This is so called quasi-TEM mode. Thus 3D electromagnetic problem reduces to 2D plane problem:

\[
\nabla \cdot \left( \varepsilon \nabla \psi \right) + \beta^2 \psi = 0, \tag{2}
\]

where \( \varepsilon \mu_0 \frac{\omega^2}{c^2} = \beta^2 + \beta_z^2 \). Applying appropriate boundary conditions the problem is solved numerically using two dimensional finite element method (2D FEM). Then one may calculate electromagnetic field distribution.
Figure 2. Comparison of effective permittivity calculation in microstrip line using 3D FEM, FDTD and quasi-static approximation ($t_i = 12$, $w = 0.5$ mm, $h = 1.5$ mm)

However, in many cases it is convenient to operate with some macroscopic parameter, rather than field distribution. We introduce effective permittivity $\varepsilon_{eff}$ relating total power in the system under consideration to power in the system with uniform filling:

$$\varepsilon_{eff} = \frac{\sum_{i=1}^{N} \int_{S_i} \left( \frac{\partial \psi_i}{\partial x} \right)^2 dxdy}{\int_{S} \left( \frac{\partial \psi_1}{\partial x} \right)^2 dxdy}$$

where $S_i$ is $i$-th domain area with permittivity $\varepsilon_i$, $S$ is line’s cross section total area, $\psi_i$ is distribution of scalar potential in regular line with $\varepsilon_i = 1$. Quasi-static approximation gives results, which coincide well with rigorous solution by 3D FEM and finite difference in time domain (FDTD) method (Fig. 2). However, at small displacements of conductor above substrate rigorous solutions faced convergence difficulties, especially FDTD method.

III. DISCUSSION

When conductor is lifted above substrate, electromagnetic filed is no longer confined in substrate. Instead, it redistributes between substrate and air in close proximity around electrode. Energy stored in the air is smaller comparing to that one in the dielectric substrate, so effective permittivity of the system decreases, as shown in Fig. 3. Effective permittivity of the line defines wavelength in the system or, equivalently, propagation constant. Thus, mutual displacement of transmission line parts results in change of propagation constant. This change is observed at device’s terminal as variable phase shift. So such lines can be used as components of microwave phase shifters, filters and so on. Important point is that displacement required for reasonable tuning is just few percent of substrate thickness (Fig. 3), so compact size and fast actuators can be employed.

On the other hand, air is practically lossless medium comparing to dielectric substrate, and part of energy, confined in close proximity to electrode, would not dissipate into heat. So, effective loss in the system would decrease too, Fig. 3.

Figure 3. Effective permittivity in near 50Ω microstrip line with micromechanical control ($w/h = 2$)

In coplanar lines effective permittivity and loss exhibit similar trends, Fig. 4.

Figure 4. Effective loss in near 50Ω coplanar line with micromechanical control ($b/a = 0.72$)

IV. CONCLUSION

Concept of effective permittivity simplifies design and analysis of tunable coplanar and microstrip lines. The concept is proven by comparison with rigorous 3D FEM and FDTD simulation. Displacement of transmission line conductor above substrate’s surface not only serves as efficient method to control propagation constant in the line, but reduces effective loss as well.

REFERENCES