Metamaterial-Based Electromagnetic and Acoustic Field Concentrators and New Physical Phenomena: Nonlinear Focusing Switching

Rapoport¹ Yu.G., Boardman² A.D., Grimalsky³ V.V., Mitchell-Thomas⁴ R., Orekhov¹ T.P.
¹Physics Faculty, Astron. and Space Phys. Dept., Kyiv Taras Shevchenko National University, Kyiv, Ukraine, e-mail: yuriy.rapoport@gmail.com
²Joule Physics Laboratory, Institute for Materials Research University of Salford, Manchester, UK, e-mail: A.D.Boardman@salford.ac.uk
³Autonomous University of Morelos (UAEM), Cuernavaca, Mexico, e-mail: v_grim@yahoo.com
⁴Queen Mary, University of London, London, UK, e-mail: rcmt17@gmail.com

Abstract – In this paper we model nonlinear electromagnetic and acoustic field/energy concentrators with new physical effect: superfocusing, and nonlinear focusing switching. For modelling such concentrators, the new method is developed, including a new form of complex geometric optics, matched to a full-wave nonlinear solution. It is shown that in the process of the nonlinear focusing switching, “hotspots” are created, when an amplitude(s) of incident beam(s) exceeds some threshold value. The proposed effect can be a foundation for many metamaterial-based applications.

Keywords – Metamaterials; superfocusing; nonlinear switching

I. INTRODUCTION

One of the most promising ways of providing the given characteristics of new electromagnetic and acoustic devices is using the principles of transformational optics (TO) [1-3]. TO allows realization, for example, of “electromagnetic or acoustic invisibility” of objects placed in an appropriate environment of inhomogeneous metamaterials, concentration of field/energy, beam splitting and rotation of polarization, subwavelength imaging, reduction of acoustic noises and other functions. We develop a new nonlinear systems for strong energy concentration, because nonlinearity provides an extra tenability and can be used, as such, as powerful tool of focusing. At the same time, our approach paves a way for developing nonlinear TO and a wide class of new tunable nonlinear devices, based on controllable metamaterials [4].

II. PHYSICAL STRUCTURE, FORMULATION OF THE PROBLEM AND THE METHOD OF MODELING

2D problem is considered and the nanostructure under consideration includes layered dielectric cylinder (Fig. 1) used for a strong focusing of incident electromagnetic/acoustic beams. In the external linear cylinder \( R_c \leq r \leq R_0 \), the inhomogeneous permittivity increases with decreasing the radius, thus ensuring light capture through graded-index means.

\[\varepsilon(r) = \begin{cases} 
\varepsilon_0, & r > R_0 \\
\varepsilon_c + i\gamma, & r < R_0 \leq r \leq R_c \\
\varepsilon_0, & r < R_0
\end{cases} \]

where \( \varepsilon_c = \varepsilon_0 R_c^2 / R_0^2 \); \( \gamma \) is the imaginary part of the dielectric permittivity. The internal region includes, in addition, saturated Kerr nonlinearity in the form \( \delta_{NL} = \alpha |E|^2 (1+\beta |E|^2) \) (see also Fig. 1), where \( \alpha, \beta \) are the constants. The inclusion of saturation (characterized by a constant \( \beta \)) is important, because we expect that the “superfocusing” would lead to a strong field concentration. Any suppositions of “nonlinearity weakness” or “slowly varying amplitudes” inside a nonlinear region are not used. A new version of “complex geometrical optics” (CGO) is developed for modeling wave/beam propagation in the external inhomogeneous linear region. There are two important features of our version of CGO: (1) spatial coordinates are real, in spite of complex eikonal and pulses, in distinction to some versions of CGO with complex coordinates [5] and (2) group velocity in weakly inhomogeneous and slightly lossy medium has a proper form, established in [6], namely \( v_g \approx \omega / \delta (\text{Re} \hat{k}) \), where \( \omega, \hat{k} \) are frequency.
and wavenumber, respectively. In the region \( R_c < r \leq R_0 \) we put \( E_z = A \exp(iS(r, \theta)) \), \( S(r, \theta) \) is complex eikonal \([5, 6]\), (the angle \( \theta \) is shown in Fig. 1), while the effective momentum components \( p_{r, \theta} \) are

\[
p_r = \frac{\partial S}{\partial r}, p_\theta = \frac{\partial S}{\partial \theta}, \quad dS = p_r dr + p_\theta d\theta	ag{2}
\]

Dispersion equation and the new equations of the proposed version of CGO have the form:

\[
\begin{align*}
D &= p_r^2 - (ip_r / r) + \left( \frac{p_\theta^2}{r^2} \right) - k_0^2 \psi(r) = 0, \quad k_0 = \omega / c \quad (3a) \\
\frac{dr}{dt} &= -\text{Re}(D_{p_r / D_{\psi}}), \quad \frac{d\theta}{dt} = -\text{Re}(D_{p_\theta / D_{\psi}}), \\
\frac{dp_r}{dt} &= [D_r / D_{\psi}] \text{Re}(D_{p_r / D_{\psi}}), \quad \frac{dp_\theta}{dt} = -k_0 \rho = \text{const} = -k_0 \rho
\end{align*}
\]

where \( D_r, D_{p_r}, D_{\psi}, D_{p_\theta} \) are the partial derivatives, \( \rho \) is the “impact distance” for the considered beam (Fig. 1). Then, inside a nonlinear cylindrical region \( r \leq R_c \), full-wave nonlinear solution is used for numerical modeling, accounting for multimode coupling. The full-wave field has the form

\[
E_z = \sum_{m=1}^{N/2} A_m(r) e^{i\phi_m} \quad (N \text{ is the number of the Fourier modes accounted for})
\]

The nonlinear dielectric constant can be presented as

\[
\delta \varepsilon_{NL}(r, \theta) = \sum_{l=-N/2}^{N/2} \delta \varepsilon_i(r) e^{-il0}, \quad \text{and the nonlinear wave equation for the amplitude of nth harmonic is:}
\]

\[
\left( \frac{1}{r} \frac{d}{dr} \right) \left( r \frac{dA_m}{dr} \right) - \left( m^2 / r^2 \right) A_m + k_0^2 \psi(r) + \sum_{l} \delta \varepsilon_l(r) A_m + \sum_{l=-N/2}^{N/2} \delta \varepsilon_l(r) (A_{m-l} - A_m) = 0
\]

Multimode coupling is included through \( \delta \varepsilon_l(r) \). Proper boundary conditions are derived on the boundary between linear and nonlinear cylinders. The proposed method, which includes the original version of CGO, full-wave strongly nonlinear electromagnetic solution, and their matching with proper boundary conditions, is applicable also for TO-based nonlinear nanomaterial field concentrators, which include both “permittivity” and “permeability”, for hyperbolic metamaterials, and wave processes in nonlinear plasma.

III. RESULTS OF THE MODELING

Sequential linear and nonlinear focusing provides a unique new “superfocusing” – extremely effective focusing of field/energy in a very small region and formation of “hot spots” on the boundary between linear and nonlinear cylinder. “Switching” (“jump”) of a nonlinear focusing position from some point inside a nonlinear cylinder (Fig. 1a) to the boundary between linear and nonlinear cylinder (Fig. 1b) is achieved, when amplitude of the incident wave exceeds some “threshold” value. Then, “hot spot” with very large amplitude is formed on the interface \( r = R_c \) (Figs. 1.2b) between linear and nonlinear regions.

Figure 2. Nonlinear switching of focusing inside the nonlinear region

To our best knowledge, this is a new strongly nonlinear effect, similar to well-known “strongly nonlinear surface waves” \([7]\) in planar waveguide. In distinction to a planar waveguide, we have “closed cylindrical structure”, and instead of surface strongly nonlinear waves (which do not exist in a linear medium), in our case, effective “boundary nonlinear resonator” is formed, containing a “hot spot”.

IV. NUMERICAL EVALUATIONS, CONCLUSIONS AND DISCUSSION

Numerical computations for Si-based concentrator (with n-doped Si in the nonlinear internal region) had been done. A possibility of strong electromagnetic (in IR range) and acoustic (in near-GHz range) field concentrations with “nonlinear focusing switching” is shown. The applications are perspective for harmonic generation, nonlinear antenna for optical fibers, acoustic noise reduction, energy harvesting, sensing. A possibilities of using for concentrators of active nanomaterials (with quantum dots), hyperbolic metamaterials and materials with nonlinear (tunable) losses are under consideration.

REFERENCES